Math 4650 - Homework # 3
Infinite Series

Part 1 - Computations

1. Calculate the value of $\displaystyle\sum_{n=2}^{\infty}\left(\frac{2}{7}\right)^{n}$

2. Calculate the value of $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{2n}$

3. Use partial fractions to show that $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}=\frac{1}{2}$

4. Use partial fractions to show that $\displaystyle\sum_{n=2}^{\infty}\frac{2}{n^2-1}=\frac{3}{2}$

Part 2 - Proofs

5. Suppose that $\displaystyle\sum_{n=1}^{\infty}a_n$ and $\displaystyle\sum_{n=1}^{\infty}b_n$ are convergent series with $\displaystyle\sum_{n=1}^{\infty}a_n = A$ and $\displaystyle\sum_{n=1}^{\infty}b_n = B$. Let α be a real number.

   (a) Prove that $\displaystyle\sum_{n=1}^{\infty}\alpha a_n$ converges to αA.

   (b) Prove that $\displaystyle\sum_{n=1}^{\infty}(a_n+b_n)$ converges to $A+B$.

6. Let (a_n) be a sequence of real numbers.
   Prove that $a_n \to 0$ if and only if $|a_n| \to 0$.

7. Determine whether the following series converge or diverge. Explain why.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{2n}{3n+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+n^2+n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{2n^3+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} (\sqrt{n+1}-\sqrt{n})$

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8. (a) Prove that if $\displaystyle\sum_{n=1}^{\infty} |a_n|$ converges then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

   (b) Give an example to show that the converse of part (a) is not true. The converse is: if $\displaystyle\sum_{n=1}^{\infty} a_n$ converges then $\displaystyle\sum_{n=1}^{\infty} |a_n|$ converges.

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9. (We used this for evaluating geometric series)
   Prove that if $-1 < r < 1$, then $\displaystyle\lim_{n\to\infty} r^n = 0$.
